Hillert & Jarl 磁気モデルの式は文献（ M.Hillert and M.Jarl, CALPHAD, 2 (1978), 227-238. ）に載っている。本書は、文献の(17)(19)式を展開する。

$$K^{\alpha} = \frac{474}{497}\left(\frac{1}{f} - 1\right)K^{\beta} \tag{9}$$

$$K^{\alpha} + 0.6K^{\beta} = \frac{675}{518}\ln({}^{0}\beta + 1) \tag{11}$$

(9)式と(11)式より、(12)式が求まる。

$$K^{\beta}\left\{\frac{474}{497}\left(\frac{1}{f} - 1\right) + \frac{3}{5}\right\} = \frac{675}{518}\ln\left({}^{0}\beta + 1\right)$$

$$\begin{aligned} K^{\beta} &= \frac{\ln\left({}^{0}\beta + 1\right)}{\frac{518}{675}\times\frac{3}{5} + \frac{518}{675}\times\frac{474}{497}\left(\frac{1}{f} - 1\right)} \\ &= \frac{\ln\left({}^{0}\beta + 1\right)}{\frac{518}{1125} + \frac{11692}{15975}\left(\frac{1}{f} - 1\right)} \end{aligned} \tag{12}$$

$$G^{\beta} = -K^{\beta}RT_{C}\left[\frac{1}{10\tau^{4}} + \frac{1}{315\tau^{14}} + \frac{1}{1500\tau^{24}}\right] \tag{17}$$

(17)式に(12)式を代入する。 Tc = T / τ の関係があり、

paramagnetic 常磁性は τ>1 であり、この自由エネルギーは

$$G^{\beta} = -R\frac{T}{\tau}\ln\left({}^{0}\beta + 1\right)\frac{\left[\frac{1}{10\tau^{4}} + \frac{1}{315\tau^{14}} + \frac{1}{1500\tau^{24}}\right]}{\frac{518}{1125} + \frac{11692}{15975}\left(\frac{1}{f} - 1\right)}$$

$$= RT\ln\left({}^{0}\beta + 1\right) * (-1) * \frac{\frac{\tau^{-5}}{10} + \frac{\tau^{-15}}{315} + \frac{\tau^{-25}}{1500}}{\frac{518}{1125} + \frac{11692}{15975}\left(\frac{1}{f} - 1\right)}$$

と展開できる。

$$G^{\alpha} = -K^{\beta} RT_C \left[\frac{79}{140} - \frac{518\tau}{1125} \right] - K^{\alpha} RT_C \left[\frac{\tau^4}{6} + \frac{\tau^{10}}{135} + \frac{\tau^{16}}{600} + \frac{71}{120} - \frac{518\tau}{675} \right] \qquad (19)$$

(19)式に(9)式を代入する。 Tc = T / τ の関係があり、

Ferromagnetic 強磁性は τ<1 であり、この自由エネルギーは

$$G^{\alpha} = -K^{\beta} RT_C \left[\frac{79}{140} - \frac{518\tau}{1125} \right] - \frac{474}{497}\left(\frac{1}{f} - 1 \right) K^{\beta} RT_C \left[\frac{\tau^4}{6} + \frac{\tau^{10}}{135} + \frac{\tau^{16}}{600} + \frac{71}{120} - \frac{518\tau}{675} \right]$$

$$= K^{\beta} RT \left[-\frac{79\tau^{-1}}{140} + \frac{518}{1125} - \frac{474}{497}\left(\frac{1}{f} - 1 \right) \left\{ \frac{\tau^3}{6} + \frac{\tau^9}{135} + \frac{\tau^{15}}{600} + \frac{71\tau^{-1}}{120} - \frac{518}{675} \right\} \right]$$

$$= K^{\beta} RT [\frac{518}{1125} - \frac{79\tau^{-1}}{140} - \frac{474}{497}\left(\frac{1}{f} - 1 \right) \left(\frac{71\tau^{-1}}{120} - \frac{518}{675} \right) - \frac{474}{497}\left(\frac{1}{f} - 1 \right) \left(\frac{\tau^3}{6} + \frac{\tau^9}{135} + \frac{\tau^{15}}{600} \right)]$$

$$= K^{\beta} RT \left[\frac{518}{1125} + \frac{11692}{15975}\left(\frac{1}{f} - 1 \right) - \frac{79\tau^{-1}}{140f} - \frac{474}{497}\left(\frac{1}{f} - 1 \right) \left(\frac{\tau^3}{6} + \frac{\tau^9}{135} + \frac{\tau^{15}}{600} \right) \right]$$

$$= \frac{\ln\left({}^{0}\beta - 1 \right)}{\frac{518}{1125} + \frac{11692}{15975}\left(\frac{1}{f} - 1 \right)} RT [\frac{518}{1125} + \frac{11692}{15975}\left(\frac{1}{f} - 1 \right) - \frac{79\tau^{-1}}{140f} - \frac{474}{497}\left(\frac{1}{f} - 1 \right) \left(\frac{\tau^3}{6} + \frac{\tau^9}{135} + \frac{\tau^{15}}{600} \right)]$$

$$= RT \ln\left({}^{0}\beta + 1 \right) \left[1 - \frac{\frac{79\tau^{-1}}{140f} + \frac{474}{497}\left(\frac{1}{f} - 1 \right) \left(\frac{\tau^3}{6} + \frac{\tau^9}{135} + \frac{\tau^{15}}{600} \right)}{\frac{518}{1125} + \frac{11692}{15975}\left(\frac{1}{f} - 1 \right)} \right]$$

と展開できる。